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# NON-EUCLIDEAN MODEL OF THE ZONAL DISINTEGRATION OF ROCKS AROUND AN UNDERGROUND WORKING

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*The non-Euclidean continuum model for the description of the stress-field distribution around underground workings with a round cross section is considered. From the physical viewpoint, the non-Euclideanness parameter determines the incompatibility of elastic deformations in a rock. It is shown that disintegration zones can be identified with the parts of the rock in which this parameter takes on the maximum values and the force discontinuity criterion for the medium holds. An analysis allows one to relate the macroscopic characteristics of zonal rock fracture around a working to the non-Euclideanness parameter.*

**Introduction.** It is known from experimental studies that a zonal periodic structure in the form of alternated regions of fractured and relatively intact rock appears around rock workings [1–5]. It is impossible to describe the occurrence of such a structure on the basis of the classical continuum model, since if one considers the problem of the stress-field distribution around a working with a round cross section upon plane deformation [6] with the stress $\sigma_\infty$ specified at infinity, then the main radial stress $\sigma_{rr}$ and the angular stress $\sigma_{\varphi\varphi}$ have extrema on the working contour and tend monotonically to $\sigma_\infty$ at infinity within the framework of the classical model. However, the experimentally observed alternation of fracture zones around a working corresponds to the occurrence of compression and tension of the rock, i.e., it shows the wavy behavior of the stress components. Shemyakin et al. [4, 5] used the plastic solution for a cylindrical cavity in a plane-deformed state to describe this behavior of the stress field. Within the framework of this model, Reva and Tropp [7] attempted to solve the boundary-value problem of the stress-field distribution around a working in a stationary state. However, "... to determine the internal boundaries, the theoretically continual body of information ... which is inaccessible under concrete conditions is needed ... ." [7, p. 129]. At the same time, it follows from the results of static experiments [8] on models from equivalent materials, which were performed with a view to studying the zonal character of fracture around workings, that, for a given material, the number of originating zones depends on the ratio of the applied stress to the strength limit of the material. Here the distance between the formed zones is approximately equal to the radius of a working.

The complexity of the development of a quantitative theory of the phenomenon of zonal disintegration is determined by the need to simulate the behavior of a medium having the properties of elastic deformation (nonfractured zones around a working) and fracture. From the physical viewpoint, the formation of fracture zones depends on the presence of microdefects in a medium which lead to the formation of macroscopic structures, in particular, a main crack repeating the shape of a working, under the action of the applied stress [8]. To describe the defects, one can use methods of the modern geometry [9], abandoning the classical hypothesis that the internal geometry of a material coincides with the geometry of the observer's Euclidean space. Here one can construct a model by increasing the number of parameters of the classical theory.

The general idea of extension of the classical model consists of the following: 1) the parameters

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that describe the non-Euclidean character of the internal geometry of a medium are introduced; 2) these parameters are related to the macroscopic characteristics of the medium; 3) the method of determining the phenomenological parameters via an analysis of experimental data is indicated. A realization of this idea is proposed in the present study. Some results of this approach were reported in [10].

**1. Transition from the Classical Model to the Non-Euclidean Model.** The mathematical model of a rock [6] in which the rock is assumed to be a plane loosed by a hole simulating a fixed round working upon triaxial compression and which is generally accepted in the mechanics of underground structures is used. The problem of the stress-field distribution around a working is given in a stationary formulation. By virtue of the polar symmetry of the problem, the equations of equilibrium have the form

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\varphi\varphi}) = 0, \qquad \sigma_{r\varphi} = 0. \tag{1.1}$$

The external forces are absent on the working contour ($r = r_0$) and they are given at infinity:

$$\sigma_{rr} = 0 \quad \text{for} \quad r = r_0, \qquad \sigma_{rr}, \sigma_{\varphi\varphi} \to \sigma_\infty \quad \text{for} \quad r \to r_\infty. \tag{1.2}$$

In the classical model, the strain components $\varepsilon_{ij}$ are reversible (elastic) and coincide with the complete strains described by the Almansi tensor $A_{ij}$ in Euler variables. Then, the compatibility conditions for strains (vanishing of the Riemann–Christoffel tensor $R_{lijk}$) are satisfied. The geometrical meaning of $R_{lijk}$ consists of the fact [11] that this tensor is an invariant characteristic of the Euclideanness of a certain set (if $R_{lijk} = 0$, one can introduce the Euclidean coordinates on this set). The fulfillment of the compatibility conditions for a rock means that its internal geometrical structure coincides with the structure of the Euclidean (external relative to the rock) space. For small strains, the compatibility conditions are called the Saint-Venant compatibility conditions and, for a plane-deformed state, they are written in the form

$$R \equiv 2\Big(\frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} - 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2}\Big) = 2\Big(\Delta \varepsilon_{ll} - \frac{\partial^2 \varepsilon_{ij}}{\partial x_i \partial x_j}\Big); \tag{1.3}$$

$$R = 0, \tag{1.4}$$

where $\Delta$ is the Laplace operator. For the classical model, the compatibility condition (1.4) is reduced to the form

$$\Delta \sigma = 0, \tag{1.5}$$

where $\sigma = \operatorname{sp} \sigma_{ij}$ is the trace of the stress tensor. Equations (1.1)–(1.5) correspond to the classical problem of the stress distribution around a working.

The formation of zones around a working is irreversible; therefore, together with the elastic-strain tensor, it is necessary to introduce the irreversible-strain tensor $\pi_{ij}$ as an additional parameter of the problem. In this case, the equations of state of the rock $\varepsilon_{ij}$ and $\pi_{ij}$ are assumed to be the thermodynamic variables. Here it is necessary to specify relations that relate $\varepsilon_{ij}$ and $\pi_{ij}$ to the Almansi tensor $A_{ij}$. The strains at which the zones form are small; therefore, the assumption that the reversible and irreversible strains are additive is used:

$$A_{ij} = \varepsilon_{ij} + \pi_{ij}. \tag{1.6}$$

Since $\varepsilon_{ij} \neq A_{ij}$, the function $R$ does not vanish. We note that, for the tensor $A_{ij}$, the representation in terms of the components of the displacement vector $u_i$, which has the form $2A_{ij} = \partial u_i/\partial x_j + \partial u_j/\partial x_i$ at small strains, always holds. Then, the compatibility conditions (1.3) and (1.4) for $\varepsilon_{ij} = A_{ij}$ are satisfied identically.

Thus, the occurrence of irreversible strains in a rock leads to the nonfulfillment of the compatibility condition for $\varepsilon_{ij}$. From the mathematical viewpoint, this means that the internal geometrical structure of the rock is non-Euclidean. Here $R$ has the meaning of a scalar curvature [11], i.e., the trace of the Ricci tensor, which completely determines the Riemann–Christoffel tensor in the three-dimensional space. Under conditions of a plane-deformed state, the scalar curvature is a single non-Euclideanness parameter of the internal rock structure. The transition from the classical model, in which $R = 0$, to the non-Euclidean model is performed by varying the latent parameter $R$.

For the non-Euclidean model, the equations of equilibrium (1.1) and the boundary conditions (1.2) remain true. The question whether Hooke's law holds in introducing the defectness parameter $R$ and which equation $R$ satisfies are answered on the basis of the principles of nonequilibrium thermodynamics. In this case, as is shown below, it is necessary to set the internal energy $U$ of a rock and the dissipation function $\mathcal{D}$.

The internal energy is assumed to be the function of entropy $s$, the reversible-strain tensor $\varepsilon_{ij}$, and the parameter $R$: $U = U(s, \varepsilon_{ij}, R)$. Since the strains $\varepsilon_{ij}$ are small, the dependence of $U$ on these variables is presented in the form of Hooke's potential. The additional contribution to this potential should take into account the dependence of $U$ on the non-Euclidean internal structure. We assume that the contribution enters additively and it is square in $R$, i.e.,

$$\rho_0 U = \frac{E}{1+\nu}\Big\{\frac{\nu}{2(1-2\nu)}\,\varepsilon_{jj}^2 + \frac{1}{2}\,\varepsilon_{ij}\varepsilon_{ij}\Big\} - \frac{q}{4}\,R^2, \qquad (1.7)$$

where $\rho_0$ is the density of the medium, $E$ is Young's modulus, $\nu$ is Poisson's ratio, and $q$ is the "fitting" parameter of the model determined from experimental data; hereinafter, summation over repeated subscripts $i$ is performed.

**2. Kinematic Relations and Equations of State.** The use of the functions $\varepsilon_{ij}$ and $R$ as variables makes it necessary to construct the transfer equations for them. During the motion, the Almansi tensor changes as follows:

$$\frac{DA_{ij}}{Dt} = \frac{dA_{ij}}{dt} + A_{il}\frac{\partial v_l}{\partial x_j} + A_{lj}\frac{\partial v_l}{\partial x_i} = \frac{1}{2}\Big(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\Big) = e_{ij}. \qquad (2.1)$$

It follows from (2.1) and (1.6) that

$$e_{ij} = \frac{D\varepsilon_{ij}}{Dt} + \frac{D\pi_{ij}}{Dt}. \qquad (2.2)$$

We introduce the source $E_{ij}$ of irreversible strains $\pi_{ij}$ [9], assuming

$$\frac{D\pi_{ij}}{Dt} = E_{ij}. \qquad (2.3)$$

From (2.2) and (2.3), we obtain the following transfer equations for the strain tensor $\varepsilon_{ij}$:

$$\frac{D\varepsilon_{ij}}{Dt} = e_{ij} - E_{ij}. \qquad (2.4)$$

In deriving a transfer equation for $R$, we take into account that, for small strains, it is not possible to distinguish the differentiation of $d/dt$ and $\partial/\partial t$; then, we have

$$\frac{dR}{dt} = -2\Big(\Delta E_{ll} - \frac{\partial^2 E_{ij}}{\partial x_i \partial x_j}\Big). \qquad (2.5)$$

Following to the standard scheme of non-equilibrium thermodynamics [12], we write the first and second laws of thermodynamics in the form

$$\rho\,\frac{dU}{dt} = -\frac{\partial J_{(q)k}}{\partial x_k} + \sigma_{ij}\,\frac{\partial v_i}{\partial x_j}, \qquad \rho\,\frac{ds}{dt} = -\frac{\partial J_{(s)k}}{\partial x_k} + \mathcal{D}, \qquad \mathcal{D} \geqslant 0. \qquad (2.6)$$

Here the functions $J_{(q)k}$ and $J_{(s)k}$ are the thermal-flux and entropy components, $\sigma_{ij}$ are the stress-tensor components, $\rho$ is the density, $s$ is the specific entropy, and $\mathcal{D}$ is a dissipation function. Along the trajectory of motion, the Gibbs identity

$$\frac{dU}{dt} = T\,\frac{ds}{dt} + \frac{\partial U}{\partial \varepsilon_{ij}}\,\frac{d\varepsilon_{ij}}{dt} + \frac{\partial U}{\partial R}\,\frac{dR}{dt}$$

holds ($T$ is the temperature). Substituting the expressions for time derivatives with respect to the internal energy and entropy from (2.6), we obtain

$$-\frac{\partial J_{(s)k}}{\partial x_k} + \mathcal{D} = -\frac{1}{T}\frac{\partial J_{(q)k}}{\partial x_k} + \frac{1}{T}\Big(\sigma_{ij}\frac{\partial v_i}{\partial x_j} - \rho\frac{\partial U}{\partial \varepsilon_{ij}}\frac{d\varepsilon_{ij}}{dt} - \rho\frac{\partial U}{\partial R}\frac{dR}{dt}\Big). \tag{2.7}$$

Using the transfer equations (2.4) and (2.5), we exclude the time derivatives with respect to $\varepsilon_{ij}$ and $R$ from the right side of (2.7); then, we have

$$\rho\frac{\partial U}{\partial \varepsilon_{ik}}\frac{d\varepsilon_{ik}}{dt} = (\delta_{ik} - 2\varepsilon_{ik})\rho\frac{\partial U}{\partial \varepsilon_{kj}}\frac{\partial v_i}{\partial x_j} - \rho E_{ij}\frac{\partial U}{\partial \varepsilon_{ij}}; \tag{2.8}$$

$$\frac{\rho}{T}\frac{\partial U}{\partial R}\frac{dR}{dt} = -\frac{J}{T}\Big(\Delta E_{ll} - \frac{\partial^2 E_{ij}}{\partial x_i \partial x_j}\Big), \qquad J \equiv 2\rho\frac{\partial U}{\partial R}. \tag{2.9}$$

We rewrite (2.9), separating the divergent contribution:

$$\frac{\rho}{T}\frac{\partial U}{\partial R}\frac{dR}{dt} = \frac{\partial}{\partial x_k}\Big(\frac{J}{T}\frac{\partial E_{ll}}{\partial x_k} - \frac{E_{ll}}{T}\frac{\partial J}{\partial x_k} - \frac{J}{T}\frac{\partial E_{kj}}{\partial x_j} + \frac{E_{kj}}{T}\frac{\partial J}{\partial x_j}\Big)$$

$$+ \frac{1}{T^2}\frac{\partial T}{\partial x_k}\Big(J\frac{\partial E_{ll}}{\partial x_k} - E_{ll}\frac{\partial J}{\partial x_k} - J\frac{\partial E_{kj}}{\partial x_j} + E_{kj}\frac{\partial J}{\partial x_j}\Big) + \frac{1}{T}\Big(E_{ll}\Delta J - E_{ij}\frac{\partial^2 J}{\partial x_i \partial x_j}\Big). \tag{2.10}$$

We substitute (2.8) and (2.10) into (2.7); as a result, we have the following relation:

$$-\frac{\partial}{\partial x_k}\Big(-\frac{J_{(q)k}}{T} + J_{(s)k} + \frac{J}{T}\frac{\partial E_{ll}}{\partial x_k} - \frac{E_{ll}}{T}\frac{\partial J}{\partial x_k} - \frac{J}{T}\frac{\partial E_{kj}}{\partial x_j} + \frac{E_{kj}}{T}\frac{\partial J}{\partial x_j}\Big) + \mathcal{D}$$

$$= \frac{1}{T}\Big[\sigma_{ij} - (\delta_{ik} - 2\varepsilon_{ik})\rho\frac{\partial U}{\partial \varepsilon_{kj}}\Big]\frac{\partial v_i}{\partial x_j} + \frac{E_{ij}}{T}\Big(\rho\frac{\partial U}{\partial \varepsilon_{ij}} + \delta_{ij}\Delta J - \frac{\partial^2 J}{\partial x_i \partial x_j}\Big)$$

$$+ \frac{1}{T^2}\frac{\partial T}{\partial x_k}\Big(-J_{(q)k} + J\frac{\partial E_{ll}}{\partial x_k} - E_{ll}\frac{\partial J}{\partial x_k} - J\frac{\partial E_{kj}}{\partial x_j} + E_{kj}\frac{\partial J}{\partial x_j}\Big). \tag{2.11}$$

In accordance with the assumptions of non-equilibrium thermodynamics, the dissipation function is represented by the bilinear form of thermodynamic forces and flows [12]: $\mathcal{D} = X_i Y_i$. The consequence of this statement and relation (2.11) are the expressions for the entropy flow and the dissipation function:

$$J_{(s)k} = \frac{J_{(q)k}}{T} - \frac{J}{T}\frac{\partial E_{ll}}{\partial x_k} + \frac{E_{ll}}{T}\frac{\partial J}{\partial x_k} + \frac{J}{T}\frac{\partial E_{kj}}{\partial x_j} - \frac{E_{kj}}{T}\frac{\partial J}{\partial x_j},$$

$$\mathcal{D} = \frac{1}{T}\Big[\sigma_{ij} - (\delta_{ik} - 2\varepsilon_{ik})\rho\frac{\partial U}{\partial \varepsilon_{kj}}\Big]\frac{\partial v_i}{\partial x_j} + \frac{E_{ij}}{T}\Big(\rho\frac{\partial U}{\partial \varepsilon_{ij}} + \delta_{ij}\Delta J - \frac{\partial^2 J}{\partial x_i \partial x_j}\Big) \tag{2.12}$$

$$+ \frac{1}{T^2}\frac{\partial T}{\partial x_k}\Big(-J_{(q)k} + J\frac{\partial E_{ll}}{\partial x_k} - E_{ll}\frac{\partial J}{\partial x_k} - J\frac{\partial E_{kj}}{\partial x_j} + E_{kj}\frac{\partial J}{\partial x_j}\Big).$$

We assume that the internal energy and the dissipation function are given; then, in accordance with (2.12), one can write the equations of state of a rock

$$\sigma_{ij} = (\delta_{ik} - 2\varepsilon_{ik})\rho\frac{\partial U}{\partial \varepsilon_{kj}}, \qquad \frac{1}{T}\Big(\rho\frac{\partial U}{\partial \varepsilon_{ij}} + \delta_{ij}\Delta J - \frac{\partial^2 J}{\partial x_i \partial x_j}\Big) = \frac{\partial \mathcal{D}}{\partial E_{ij}}. \tag{2.13}$$

For a thermal flux, we accept the approximation of linear relations

$$-J_{(q)k} + J\frac{\partial E_{ll}}{\partial x_k} - E_{ll}\frac{\partial J}{\partial x_k} - J\frac{\partial E_{kj}}{\partial x_j} + E_{kj}\frac{\partial J}{\partial x_j} = \lambda\frac{\partial T}{\partial x_k} \quad (\lambda \geqslant 0), \tag{2.14}$$

where $\lambda$ is the phenomenological parameter.

In the approximation of small strains, we assume that $\rho = \rho_0$. Substituting (1.7) into (2.13), we obtain

$$\sigma_{ij} = \frac{E}{1+\nu}\Big(\varepsilon_{ij} + \delta_{ij}\frac{\nu}{1-2\nu}I_1\Big), \qquad I_1 = \varepsilon_{ll}; \tag{2.15}$$

$$\frac{1}{T}\Big(\sigma_{ij} - q\delta_{ij}\Delta R + q\,\frac{\partial^2 R}{\partial x_i \partial x_j}\Big) = \frac{\partial \mathcal{D}}{\partial E_{ij}}, \tag{2.16}$$

where $I_1$ is the first invariant of the elastic-strain tensor. It follows from (2.15) that, upon parametrization of the internal energy specified by relation (1.7), the stress and reversible-strain components are related by Hooke's classical law.

**3. Calculation of the Defectness Parameter *R*.** Function $R$ satisfies Eq. (2.5). In a stationary state, we have $dR/dt = 0$ and

$$\Delta E_{ll} - \frac{\partial^2 E_{ij}}{\partial x_i \partial x_j} = 0. \tag{3.1}$$

The sources $E_{ij}$ are determined in terms of the dissipation function $\mathcal{D}$ in accordance with (2.16). To ensure the nonnegative $\mathcal{D}$, we specify the dependence between the source $E_{ij}$ of irreversible strains and the thermodynamic forces in the form

$$E_{ij} = \xi\Big(\sigma_{ij} - q\delta_{ij}\Delta R + q\,\frac{\partial^2 R}{\partial x_i \partial x_j}\Big) \qquad (\xi \geqslant 0). \tag{3.2}$$

Substituting (3.2) into (3.1), we have

$$2q\Delta^2 R - \Delta\sigma = 0, \qquad \sigma = \sigma_{ll}. \tag{3.3}$$

The quantity $\sigma$ is found from the equation of state (2.15): $\sigma = EI_1/(1-2\nu)$. The relation between the first invariant $I_1$ and $R$ is determined by relation (1.3). Substituting

$$\varepsilon_{ij} = [(1+\nu)\sigma_{ij} - \delta_{ij}\nu\sigma]/E \tag{3.4}$$

into (1.3), we obtain

$$R = 2\Big(\Delta I_1 + \frac{\nu}{E}\,\Delta\sigma\Big) = 2\Big(\frac{1-2\nu}{E}\,\Delta\sigma + \frac{\nu}{E}\,\Delta\sigma\Big) = \frac{2(1-\nu)}{E}\,\Delta\sigma. \tag{3.5}$$

In the limiting case of the classical model, the function $R = 0$, and Eq. (3.5) is reduced to the compatibility equation (1.5). From (3.3) and (3.5), follows the equation

$$\Delta^2 R - \gamma^2 R = 0, \quad \text{where} \quad \gamma^2 = E/[4q(1-\nu)]. \tag{3.6}$$

To write the boundary conditions for $R$, we first consider the thermal flux $J_{(q)k}$ (2.14), assuming that $T = \text{const}$. The nonzero contribution to $J_{(q)k}$ coincides with the density of the flow of defects in the rock. Since the defects do not leave the bounds of the working ($r = r_0$), the normal component of the flow vector should vanish for $r = r_0$, i.e.,

$$(\boldsymbol{n}, \boldsymbol{J}_{(q)})\Big|_{r=r_0} = 0. \tag{3.7}$$

As $r \to \infty$, the function $R$ should satisfy the natural, from the physical viewpoint, requirement for a decrease at infinity. It is noteworthy that Eq. (3.6) and the boundary conditions formulated for it hold upon plane and spatial deformation.

For the case of plane deformation considered, the dependence on the polar angle is absent; then, the function $R(r)$ satisfies the equation

$$\Big(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\,\frac{\partial}{\partial r}\Big)^2 R = \gamma^2 R.$$

This is a fourth-order linear differential equation, and its solution which decreases as $r \to \infty$ is written in the form

$$R(r) = aJ_0(\sqrt{\gamma}r) + bN_0(\sqrt{\gamma}r) + cK_0(\sqrt{\gamma}r), \tag{3.8}$$

where $J_0$, $N_0$, and $K_0$ are zero-order Bessel, Neumann, and MacDonald cylindrical functions, respectively. We use the following boundary conditions (3.7) in the plane case:

$$\Big[\Big(J\frac{\partial E_{22}}{\partial x_1} - E_{22}\frac{\partial J}{\partial x_1} - J\frac{\partial E_{12}}{\partial x_2} + E_{12}\frac{\partial J}{\partial x_2}\Big)\cos\varphi + \Big(J\frac{\partial E_{11}}{\partial x_2} - E_{11}\frac{\partial J}{\partial x_2} - J\frac{\partial E_{12}}{\partial x_1} + E_{12}\frac{\partial J}{\partial x_1}\Big)\sin\varphi\Big]_{r=r_0} = 0. \quad (3.9)$$

We pass to the polar coordinates in (3.9) according to the formulas

$$\frac{\partial}{\partial x_1} = \cos\varphi\,\frac{\partial}{\partial r} - \frac{\sin\varphi}{r}\,\frac{\partial}{\partial\varphi}, \qquad \frac{\partial}{\partial x_2} = \sin\varphi\,\frac{\partial}{\partial r} + \frac{\cos\varphi}{r}\,\frac{\partial}{\partial\varphi},$$

$$E_{rr} = E_{11}\cos^2\varphi + E_{22}\sin^2\varphi + E_{12}\sin 2\varphi, \qquad E_{\varphi\varphi} = E_{11}\sin^2\varphi + E_{22}\cos^2\varphi - E_{12}\sin 2\varphi,$$

$$E_{r\varphi} = -((E_{11} - E_{22})/2)\sin 2\varphi + E_{12}\cos 2\varphi.$$

Omitting straightforward calculations, we obtain

$$\Big[J\Big(\frac{\partial E_{\varphi\varphi}}{\partial r} - \frac{1}{r}\,\frac{\partial E_{r\varphi}}{\partial\varphi} - \frac{E_{rr} - E_{\varphi\varphi}}{r}\Big) - E_{\varphi\varphi}\,\frac{\partial J}{\partial r} + \frac{E_{r\varphi}}{r}\,\frac{\partial J}{\partial\varphi}\Big]_{r=r_0} = 0.$$

Since the sources are assumed to be independent, we have $J|_{r=r_0} = 0$, $(\partial J/\partial r)|_{r=r_0} = 0$, and $(\partial J/\partial\varphi)|_{r=r_0} = 0$. Using the explicit form of the potential (1.7) and taking into account the definition of $J$ (2.9), we have the following boundary conditions for the function $R$:

$$R\Big|_{r=r_0} = 0, \qquad \frac{\partial R}{\partial r}\Big|_{r=r_0} = 0. \qquad (3.10)$$

Substituting (3.8) into (3.10), we obtain an algebraic inhomogeneous system of equations for determination of the coefficients $a$ and $b$ in terms of $c$ whose determinant coincides with the Wronskian of the linearly independent solutions $J_0$ and $N_0$, which guarantees its unique solvability:

$$a = (c/2)\pi\sqrt{\gamma}r_0[K_0(\sqrt{\gamma}r_0)N_1(\sqrt{\gamma}r_0) - K_1(\sqrt{\gamma}r_0)N_0(\sqrt{\gamma}r_0)],$$

$$b = -(c/2)\pi\sqrt{\gamma}r_0[K_0(\sqrt{\gamma}r_0)J_1(\sqrt{\gamma}r_0) - K_1(\sqrt{\gamma}r_0)J_0(\sqrt{\gamma}r_0)].$$

**4. Calculation of Stress Components.** For the case of plane deformation considered, the components $\varepsilon_{zz}$, $\varepsilon_{z\varphi}$, and $\varepsilon_{zr}$ are zero. Here the stress $\sigma_{zz} \neq 0$ and, as follows from (3.4), it is determined from the relation $(1+\nu)\sigma_{zz} = \nu\sigma$, where $\sigma = \sigma_{zz} + \sigma_{rr} + \sigma_{\varphi\varphi}$. Hence, we find that $\sigma_{zz} = \nu(\sigma_{rr} + \sigma_{\varphi\varphi})$ and, with allowance for (1.2), we have $\sigma \to 2(1+\nu)\sigma_\infty$ as $r \to \infty$.

The function $\sigma$ satisfies Eq. (3.5) with a known function $R$ and the solution of this equation is given by the formula

$$\sigma = -\frac{E}{2\gamma(1-\nu)}\,[aJ_0(\sqrt{\gamma}r) + bN_0(\sqrt{\gamma}r) - cK_0(\sqrt{\gamma}r)] + 2(1+\nu)\sigma_\infty.$$

We substitute $\sigma_{\varphi\varphi} = \sigma/(1+\nu) - \sigma_{rr}$ into the equation of equilibrium (1.1):

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{2\sigma_{rr}}{r} = \frac{\sigma}{(1+\nu)r}. \qquad (4.1)$$

Integrating (4.1), we use the formulas for differentiation of cylindrical functions

$$\frac{d}{dr}\,[rJ_1(r)] = rJ_0(r), \quad \frac{d}{dr}\,[rN_1(r)] = rN_0(r), \quad \frac{d}{dr}\,[rK_1(r)] = -rK_0(r).$$

After appropriate calculations, we obtain the expressions for the stress components:

$$\sigma_{rr} = \sigma_\infty\Big(1 - \frac{r_0^2}{r^2}\Big) - \frac{E}{2\gamma^{3/2}(1-\nu^2)r}\,[aJ_1(\sqrt{\gamma}r) + bN_1(\sqrt{\gamma}r) + cK_1(\sqrt{\gamma}r)],$$

$$\sigma_{\varphi\varphi} = \sigma_\infty\Big(1 + \frac{r_0^2}{r^2}\Big) + \frac{E}{2\gamma^{3/2}(1-\nu^2)r}\,[aJ_1(\sqrt{\gamma}r) + bN_1(\sqrt{\gamma}r) + cK_1(\sqrt{\gamma}r)] \qquad (4.2)$$

$$- \frac{E}{2\gamma(1-\nu^2)}\,[aJ_0(\sqrt{\gamma}r) + bN_0(\sqrt{\gamma}r) - cK_0(\sqrt{\gamma}r)].$$

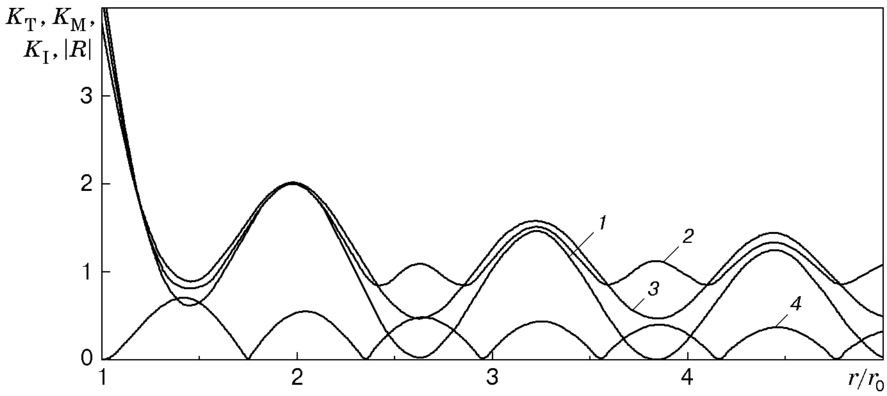


Fig. 1

**5. Localization of Disintegration Zones.** Knowing the stress components and the function $R$, it is necessary to separate the rock regions that correspond to disintegration zones. Since $R$ is a characteristic of strain incompatibility that determines the discontinuity of a medium in these zones, the maxima $|R|$ should be identified with fracture regions of these zones. However, as the experiment shows, disintegration zones appear when the stresses in a material reach a certain critical value. From the physical viewpoint, this means that it is necessary to use a force criterion, whose fulfillment in a selected region corresponds to the occurrence of a zone. As such criteria, we use the Mises, Treska, and Ishlinskii conditions. We introduce functions that correspond to these conditions:

$$K_{\rm M} = A\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2},$$

$$K_{\rm T} = B \max |\sigma_i - \sigma_j|, \qquad K_{\rm I} = C \max |\sigma_i - \sigma|.$$

Here $\sigma_j$ are the principal-stress components and $A$, $B$, and $C$ are the "fitting" parameters. Then, the disintegration zones should be identified with a region in the rock in which the functions $K_{\rm M}$, $K_{\rm T}$, and $K_{\rm I}$ reach the maximum values simultaneously. To determine the parameters $A$, $B$, $C$, $\gamma$, and $c$, it suffices to have data on the site of occurrence of the first disintegration zone. One can predict the position of other zones and the stress necessary for their occurrence within the framework of the above-described model.

The model parameters were chosen as follows. The parameter $c$ is chosen in such a way that the classical contribution to the solution (4.2) and the additional contribution have the same order of magnitude for $r > r_0$. Then, it follows from the asymptotical behavior of the functions that $c$ is a quantity of the order of $\exp(\sqrt{\gamma} r_0)/r_0^2$. The parameter $\gamma$ depends on the radius of a working $r_0$ and it is chosen in such a way that the period of the function $R$ is equal to the distance from the edge of the working to the first disintegration zone. Knowing the value of the stress $\sigma_\infty^*$ at which the first fracture zone forms, we choose the constants $A$, $B$, and $C$ in such a way that the functions $K_{\rm T}$, $K_{\rm M}$, and $K_{\rm I}$ reach a certain value $K^*$ in the first disintegration zone for $\sigma_\infty = \sigma_\infty^*$.

The numerical calculation were performed for the model of a working [8] with $r_0 = 0.07$ m, and the values of the physical constants of equivalent materials were as follows: $\sigma_\infty^* = 1.1$ MPa, $E = 150$ MPa, and $\nu = 0.15$. Here the following "fitting" parameters were used: $\sqrt{\gamma} r_0 = 5.2$, $c = 18{,}620$ m$^{-2}$, $K^* = 2$ MPa, $A = 0.75$, $B = 1$, and $C = 1.5$. Calculation results are shown in Fig. 1. The ratio of the distance $r$ from the center of the working (coordinate origin) to its radius $r_0$ is laid off as abscissa, and the values of the criterion functions $K_{\rm T}$, $K_{\rm M}$, and $K_{\rm I}$ are laid off as ordinate; in addition, the non-Euclideanness parameter $|R|$ is laid off along the Y axis after renormalization, because $|R|$ has a different dimensionality compared to the functions $K_{\rm T}$, $K_{\rm M}$, and $K_{\rm I}$. Curve 1 corresponds to the function $K_{\rm T}$, curve 2 to $K_{\rm M}$, curve 3 to $K_{\rm I}$, and curve 4 to $|R|$. According to the adopted hypothesis, the first disintegration zone corresponds to a simultaneous attainment of the maxima for $r = 2r_0$ by all the criterion functions. As is seen in Fig. 1, the second and third disintegration zones should occur for $r = 3.2r_0$ and $r = 4.4r_0$. This conclusion coincides with results of the experiments [8] performed on models from equivalent materials.

**6. Discussion of Results.** The above-considered variant of generalization of the classical theory with a transition from an Euclidean to a non-Euclidean internal geometrical structure of a rock allows one to connect the geometrical characteristics with the macroscopic parameters of the zonal disintegration of rocks around a working within the framework of the traditional formalism of non-equilibrium thermodynamics. We make a few comments on the assumptions adopted in the proposed model.

We note that in choosing the dependence of the internal energy (1.7) on the parameter $R$, it is necessary to take into account an additional contribution of the form $\gamma_2 R I_1$, which corresponds to the energy of interaction of a defective structure with the field of elastic deformations ($\gamma_2$ is an additional parameter of the model). However, as a preliminary analysis shows, the new special functions in the expressions for the stress components and the defectness parameter $R$ do not appear, and the solution is periodic. The presence of the additional "fitting" parameter $\gamma_2$ allows one to coordinate the modeling results and the experimental data more exactly.

In deriving Eq. (2.5), the difference between the differentiation of $d/dt$ and $\partial/\partial t$ has been ignored because of the smallness of strains. Nevertheless, the operator of the full derivative remains on the left side of (2.5) in the case of finite deformations as well. To substantiate this statement, it is necessary to use the equation for the Riemann–Christoffel tensor $R_{lijq}$ in the case of complete strains [9]; then, using the definition of $R = g^{jl} g^{iq} R_{lijq}$ [11], one should obtain Eq. (2.5) ($g^{jl}$ and $g^{iq}$ are the elements of the inverse matrix of an internal metric tensor).

Relation (2.5) and conditions (3.10) also hold for a spherical working. If one considers the stationary problem of the stress-field distribution for it, as calculations show, the periodic character of the behavior of the main stress components is determined by the linear combination of $\sin(\sqrt{\gamma} r)$, $\cos(\sqrt{\gamma} r)$, and $\exp(-\sqrt{\gamma} r)$ with coefficients in the form of polynomials in $1/r$. We note that in the spatial case, the Riemann–Christoffel tensor has two additional invariants which should be included into the model by taking into account the different orientation of defective rock structures, in addition to the scalar curvature $R$.

In this study, a periodic stationary structure around a working has been given. However, complicated questions arise in describing the way of its formations. In particular, in a real rock, there are microheterogeneities (the occurrence of which depends on concrete conditions for the formation of a rock) determining the magnitude of strain incompatibility in the initial state. Here the disintegration zones are located with a periodicity determined by the radius of a working. An analysis of the possibilities of realizing the initial conditions calls for the concretization of the dissipative characteristics of the material to be examined and an additional experimental study of deformation fields with a different level of resolution in measurements.

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